

# Color Transparent GPDs?

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## Abstract

The relation between GPD's and color transparency is explored. The discovery of color transparency in pionic diffractive dissociation reactions allows us to make specific predictions for the behavior of the pion generalized parton distribution, and provide a further test of any model of the pion form factor.

## 1 Introduction

Color transparency is the vanishing of initial and final state interactions, predicted by QCD to occur in coherent nuclear processes for large values of transferred momentum[1, 2, 3, 4, 5]. For such processes, the interactions of a color singlet object are controlled by the color electric dipole moment which is small if the quark-gluon constituents are close together.

Consider, for example, the  $(e, e'p)$  reaction for quasi-elastic kinematics. Color transparency may occur when those components of the hadron wave function that dominate the proton form factor at large  $Q^2$  have a small transverse size. This is indeed the case if the form factor at large momentum transfer is correctly described by perturbative Quantum Chromodynamics, pQCD. However, the power law behavior obtained from pQCD arguments can also be obtained from the Feynman mechanism in which the active quark carries almost all the  $p^+$  momentum of the hadron. The spectator quarks are then “wee partons” which have no direction and hence it is very easy to turn them around. The wee parton cloud is not expected to be of small transverse size and therefore such components of the wave function would interact strongly as the emitted object moves out of the nucleus. Then color transparency would not be observed. Color transparency is not a necessary result even if a small-sized configuration dominates the form factor[4]. The configuration must remain of small size during the time required for escape from the nucleus, and this requires high energies.

Dominance of small-size, or point-like configurations, at a high momentum transfers is a necessary condition on the existence of color transparency, and it is worthwhile to see if this can be related to other features of hadronic wave functions. Here we shall examine generalized parton distributions (GPDs) which have the amazing property of providing a decomposition of the form factor with respect

to the light-cone momentum of the active quark

$$F_1(Q^2) = \int dx H(x, 0, Q^2). \quad (1)$$

Eq. (1) implies that if, we know the function  $H(x, 0, Q^2)$  at large  $Q^2$ , then we also know the important regions of  $x$  for the form factor. This knowledge should enable us to differentiate between suggested mechanisms that drive form factors at large  $Q^2$  and should ultimately allow us to understand whether or not color transparency may occur for the proton.

Providing a logical link between GPDs and color transparency would allow physicists to answer some general questions. Can the observation of color transparency determine any features of GPDs? Conversely, could a measurement of a GPD for a given system enable us to predict the ability to observe the effects of color transparency?

To answer these questions it is necessary to review some background information about color transparency, CT. There are three requirements for CT to occur[6] in coherent reactions:

- small color neutral objects do not interact
- high momentum transfer (semi) exclusive reactions proceed through configurations of small size, that we call point like configurations, PLC. This depends on the process occurring with a high enough momentum transfer.
- the PLC must move quickly enough through the nucleus to escape prior to expansion (which is inevitable). This depends on the PLC having large energy since the time dilation factor is the energy divided by a mass.

The second requirement addresses the most interesting aspect of color transparency physics: the necessity of the formation of a PLC for a high momentum transfer coherent reaction to proceed, and is our focus. The formation of a PLC is a natural occurrence in PQCD[1, 2], but may (and perhaps can be expected to) arise in strong QCD[3, 7].

An operational method to test whether or not a given model for a hadron allows the formation of PLC was devised[3, 7]. Let's use a schematic notation to understand the basic idea. Suppose one has an initial hadron  $|H\rangle$  that is subject to a high momentum transfer reaction. Let the hard operator that brings in the high momentum be denoted as  $T_H(Q^2)$ . The operator  $T_H(Q^2)$  arises solely from the gluon exchanges within the hadron  $H$ , and depend only on the nature of the target. The resulting wave packet is denoted as  $T_H(Q^2)|H\rangle$ , which or may or may not be a  $|PLC\rangle$ . Suppose this object propagates through the nucleus, and is detected, moving with the transferred momentum. Then the scattering amplitude is given by

$$\mathcal{M} = \langle H(Q^2) | (1 + T_S G_0) T_H(Q^2) | H \rangle, \quad (2)$$

where the operator  $G_0$  represents the free propagation and  $T_S$  denotes the soft final state interaction, to all orders of interaction ( $T_S = U_S(1 + G_0 T_S)$ ), with the nucleus. If  $T_H(Q^2)|H\rangle$  is a PLC (which is a wave packet and not an eigenstate of the Hamiltonian) the inevitable action of the propagation is to cause the object to expand. Suppose the energy is large enough for the packet to remain in a fixed configuration as it moves through the nucleus. Then the influence of the final state interactions is determined by  $\langle H(Q^2) | U_S T_H(Q^2) | H \rangle$ . If the struck object really is small, the scattering amplitude for the interaction of an energetic, colorless wave packet of small transverse size, characterized by a length  $b$ , is proportional to  $b^2$  (times logarithmic corrections)[8]-[11]. This dependence can be thought qualitatively as arising from the action of two color dipole operators. Then

$$U_S \propto b^2, \quad (3)$$

and the relevant matrix element that determines the importance of final state interactions is  $\langle H(Q^2) | b^2 T_H(Q^2) | H \rangle$ , and the strength of the second term of Eqn. (2) compared to the first term is determined by the ratio  $\langle H(Q^2) | b^2 T_H(Q^2) | H \rangle / \langle H(Q^2) | T_H(Q^2) | H \rangle$ .

The denominator is simply the hadronic form factor,  $F(Q^2)$ , so that we may define an effective size  $b^2(Q^2)$ :

$$b^2(Q^2) = \frac{\langle H(Q^2) | b^2 T_H(Q^2) | H \rangle}{F(Q^2)}. \quad (4)$$

CT can only occur if  $b^2(Q^2)$  is much smaller than the mean square radius of the hadron. A large magnitude of  $b^2(Q^2)$  causes the final state interactions to be large so that CT would not occur. Note that  $b^2(Q^2)$  depends on the full eigenstate, and as an off diagonal matrix element, is not positive definite. The term  $b^2(Q^2)$  takes its meaning as an effective size from its operational importance in determining whether or not final state interactions will occur.

We want to use the important fact that color transparency has been observed in diffractive dissociation of high energy pions[12, 13]. The cross sections were found to scale roughly as  $A^\alpha$  (with  $\alpha$  varying between 4/3 and 5/3) instead of the usual  $A^{2/3}$  dependence typical of diffractive cross sections. The consequence is that CT causes the ratio of the cross sections for Platinum to Carbon targets to increase by a spectacular factor of 7[14]. Indeed, the relative simplicity of the pionic wave function, the availability of a high energy pion beam, and predicted unusual  $A$ -dependence [15] made this reaction ideal for studies of color transparency.

Here is an outline of the remainder of this paper. The relation between color transparency and GPDs is explored in Sect. 2 by considering the effective size[3, 7] of an emitted particle using an impact representation[16]. That the di-jet measurements are shown to imply a very small value of  $b^2(Q^2)$  is shown in Sect. 3. As a result, most of the remainder is concerned with the pion. The effective size is computed for a variety of models of GPDs, ranging from very simple factorized forms to more complicated wave function models that include the effects of quark spin, in Sect. 4. These studies allow us to arrive at precise statements between GPDs and the existence of color transparency that are summarized in Sect. 5. If we use the experimental observation of color transparency[12, 13], then some models may be ruled out, and the behavior of the GPD at large momentum fraction  $x$  is determined.

## 2 Master Formula

Since CT is related to configurations that are small in position space, it is very useful to start from an impact parameter description [16]. The distribution of partons in impact parameter space can be easily obtained from generalized parton distributions by means of a Fourier transform, yielding

$$\mathcal{H}(x, \mathbf{B}) = \int \frac{d^2 Q}{(2\pi)^2} e^{i\mathbf{B} \cdot \mathbf{Q}} H(x, 0, Q^2). \quad (5)$$

Together with Eq. (1) this provides an impact parameter space representation for the form factor [17]

$$F(Q^2) = \int dx H(x, 0, Q^2) = \int dx d^2 B \mathcal{H}(x, \mathbf{B}) e^{-i\mathbf{B} \cdot \mathbf{Q}} \quad (6)$$

The impact parameter  $\mathbf{B}$  is measured relative to the transverse center of momentum of the hadron. This impact parameter can be easily related to the variable  $\mathbf{b}$  which measures the distance between the active quark and the center of momentum of all the spectators

$$\mathbf{B} = (1 - x)\mathbf{b}. \quad (7)$$

At large  $Q^2$  one expects that the form factor is expected to be dominated by the valence component of the wave function. Therefore in the case of meson form factors  $\mathbf{b} = \mathbf{B}/(1 - x)$  is a direct measure of the size of the system. In a nucleon (and also in a non-valence configuration of a meson), knowledge of  $\mathbf{b} = \mathbf{B}/(1 - x)$  still provides a lower bound on the overall size of the system. It is thus useful to define a  $Q^2$ -dependent size  $b^2(Q^2)$ . The interaction of a struck nucleon with the surrounding medium

depends on the size in a direction transverse to that of the large momentum  $\mathbf{Q}$ . Consequently we take  $\mathbf{Q}$  to lie in the  $x$ -direction and study the size of the  $y$  component of  $\mathbf{B}$ . Under the condition that  $\mathcal{H}(x, \mathbf{B}) = \mathcal{H}(x, \mathbf{B} \cdot \mathbf{B}) = \mathcal{H}(x, B)$  it is straightforward to obtain[7]

$$\begin{aligned} b^2(Q^2)F(Q^2) &= \frac{1}{2} \int dx d^2 B \frac{B_y^2}{(1-x)^2} \mathcal{H}(x, \mathbf{B}) e^{-iB_x Q} \\ &= - \left( \frac{\partial}{\partial Q^2} \right) \int dx d^2 B \frac{1}{(1-x)^2} \mathcal{H}(x, \mathbf{B}) e^{-i\mathbf{B} \cdot \mathbf{Q}}. \end{aligned} \quad (8)$$

Using Eq. (6), this implies an interesting relation between GPDs and the effective size of a hadron at large  $Q^2$

$$b^2(Q^2) = - \frac{\left( \frac{\partial}{\partial Q^2} \right) \int \frac{dx}{(1-x)^2} H(x, 0, Q^2)}{\int_0^1 dx H(x, 0, Q^2)}. \quad (9)$$

The rest of the paper will be devoted to discussing the implications of this result for the behavior of GPDs.

Notice the factor  $\frac{1}{(1-x)^2}$  in Eq. (9), which appears because the distance  $\mathbf{b}$  between the active quark and the spectator(s) is larger than the impact parameter  $\mathbf{B}$  (the distance to the center of momentum) by a factor  $1/(1-x)$ . This factor diverges for  $x \rightarrow 1$ , allowing a hadron of ordinary size to support a large form factor if the integrand of Eq. (6) is dominated contributions from the region with momentum fraction  $x$  near 1. In particular, the main issue is that the variable conjugate to  $Q$  is the impact parameter which needs to be divided by  $1-x$  in order to obtain the separation between the active quark and the spectator(s) — which is a better measure for the “size” of the hadron than the impact parameter.

A necessary condition for color transparency to occur is that

$$\lim_{Q \rightarrow \infty} b^2(Q^2) = 0. \quad (10)$$

This condition is not sufficient because the distance from the active quark to the center of momentum of the spectators only provides a lower bound on the size of the configuration if there is more than one spectator parton. Furthermore  $\int d^2 B B_y^2 \mathcal{H}(x, \mathbf{B}) e^{-iB_x Q}$  is strictly speaking not positive definite, although it turns out to be positive for commonly used parametrizations of GPDs at large  $Q$  — at least for those where  $\mathcal{H}(x, \mathbf{B})$  satisfies the usual positivity constraints [18].

Nevertheless, since Eq. (10) is a necessary condition for small size configurations, the existence of color transparency would constrain the possible analytic behavior of GPDs as we will discuss in the rest of the paper.

### 3 Color Transparency in Di-jet Production and $b^2(Q^2)$

To utilize the experimental discovery, it is necessary to relate di-jet production to color transparency in the  $(e, e'\pi)$  reaction, which is closely related to the physics of the pion form factor discussed in Eqs. (2-4). In this connection, it is useful to explicitly present the coordinate-space representation for the putative PLC:

$$\langle x, \mathbf{b} | T_H(Q^2) | \pi \rangle = e^{i\mathbf{q} \cdot \mathbf{b}(1-x)} \psi_\pi(x, \mathbf{b}) = \langle x, \mathbf{b} | \text{PLC}(Q^2) \rangle. \quad (11)$$

In coherent nuclear di-jet production the incident pion beam is changed into a state consisting of a  $q, \bar{q}$  pair moving at high relative transverse momentum ( $\kappa$ , but with a total momentum close to  $\mathbf{P}_\pi$ ). The quark has longitudinal momentum  $x\mathbf{P}_\pi$ , and the cross section is dominated by the region  $x \approx 1/2$ . Each quark becomes a hadronic jet at distances outside the target.

The interactions with the nuclear target must be of low momentum transfer because the target is not excited, so that the di-jet cross section depends on the matrix element[15]

$$\mathcal{M}^{\text{JJ}} = \langle x, \boldsymbol{\kappa} | (1 + T_S G_0) U_S(Q^2) | \pi \rangle, \quad (12)$$

Where  $\boldsymbol{\kappa} = (1 - x)\mathbf{k}_1 - x\mathbf{k}_2$  and  $\mathbf{k}_1, \mathbf{k}_2$  represent the momenta of  $q\bar{q}$  pair produced at high relative momentum. Each of these objects produces a jet. For a coherent process to occur, the sum of their longitudinal momenta must be close to the pion beam momentum, the sum of the transverse momenta must be very small.

The observation of color transparency is tells us that the effects of final and initial state interactions are negligible, so that the ratio

$$b_{\text{JJ}}^2(Q^2) = \frac{\langle x, \boldsymbol{\kappa} | b^2 U_S(Q^2) | \pi \rangle}{\langle x, \boldsymbol{\kappa} | U_S(Q^2) | \pi \rangle} = \frac{\langle x, \boldsymbol{\kappa} | b^4 | \pi \rangle}{\langle x, \boldsymbol{\kappa} | b^2 | \pi \rangle}, \quad (13)$$

in which the second equation is obtained using Eq. (3), must be much smaller in magnitude than the mean square radius of the pion. If this ratio were not small, the effects of initial and final state interactions would cause the  $A$  dependence to differ drastically from what was observed.

The small nature of  $b_{\text{JJ}}^2(Q^2)$  means that  $\langle x, \boldsymbol{\kappa} | U_S | \pi \rangle$  acts as a PLC, and that

$$\mathcal{M}^{\text{JJ}} \approx \langle x, \boldsymbol{\kappa} | U_S | \pi \rangle \propto \int d^2b e^{-i\boldsymbol{\kappa} \cdot \mathbf{b}} b^2 \psi_\pi(x, \mathbf{b}) = \int d^2b b^2 \langle x, \mathbf{b} | \text{PLC}(Q^2) \rangle, \quad (14)$$

where here

$$Q^2 = \left( \frac{\boldsymbol{\kappa}}{1 - x} \right)^2 \approx 4\boldsymbol{\kappa}^2. \quad (15)$$

Eqn. (14) shows the strong connection between the physics of the form factor (which is the integral of Eqn. (14) without the factor of  $b^2$ ) and that of di-jet production. We may integrate Eqn. (14) by parts to obtain

$$\mathcal{M}^{\text{JJ}} \propto \nabla_\kappa^2 \int d^2b \langle x, \mathbf{b} | \text{PLC}(Q^2) \rangle \quad (16)$$

$$\propto \frac{1}{\kappa^2} \int d^2b \langle x, \mathbf{b} | \text{PLC}(Q^2) \rangle, \quad (17)$$

in which the power law falloff of the cross section with  $\kappa^2$  was used to obtain the result (17). Eqs. (11) and (17) tell us that because  $|\text{PLC}(Q^2)\rangle$  acts as small in the di-jet production reaction,  $b^2(Q^2)$  must be small, for the kinematics of (15). We wish to explore the consequences of that smallness for pionic GPDs, and as a result much of this paper is concerned with pionic models.

## 4 The pion form factor

Empirically, it has been established that the pion form factor falls like

$$F(Q^2) = \frac{c}{Q^2} \quad (18)$$

at large  $Q^2$  (modulo logarithmic corrections). There are infinitely many different possible ansätze for  $H(x, 0, Q^2)$  that are consistent with this behavior and therefore we need to restrict ourselves to those classes of functions that are most commonly used to parameterize  $H(x, 0, Q^2)$ .

## 4.1 Factorisable Models

The first class of models that we consider are such that the large  $Q^2$  behavior factorizes from the  $x$ -dependence

$$H(x, 0, Q^2) \sim f(x)F(Q^2) \quad \text{for } Q^2 \rightarrow \infty. \quad (19)$$

This class covers the whole set of models, where  $H(x, 0, Q^2)$  already has a  $1/Q^2$  behavior. An example is provided by the “asymptotic” pion wave function, which yields

$$H(x, 0, Q^2) \sim \frac{x^2(1-x)^2}{Q^2} \quad \text{for } Q^2 \rightarrow \infty. \quad (20)$$

Clearly,  $b^2(Q^2) \sim \frac{1}{Q^2}$  for all these examples, provided  $\int dx \frac{f(x)}{(1-x)^2}$  converges. The pathetic case, where  $\int dx \frac{f(x)}{(1-x)^2}$  diverges, corresponds to a hadron that has an infinite size, and we therefore do not discuss this possibility. As a result, we find color transparency for all GPDs, where the large  $Q^2$  behavior factorizes (19).

## 4.2 Exponential Models

Another important class of models is the one where  $H(x, 0, Q^2)$  at fixed  $x$  falls faster than  $1/Q^2$ , e.g. a Gaussian model[19] where

$$H(x, 0, Q^2) = f(x) \exp \left( -a^2 Q^2 \frac{1-x}{x} \right). \quad (21)$$

In this class of models, obtaining the  $Q^2 \rightarrow \infty$  behavior of both  $F(Q^2)$  and  $b^2(Q^2)$ , depends on the crucial region in the vicinity of  $x = 1$ . This is why all factors of  $x$  are irrelevant for the discussion and we drop them for simplicity, by considering the class of functions

$$H(x, 0, Q^2) \xrightarrow{x \rightarrow 1} (1-x)^{m-1} \exp \left( -a(1-x)^n Q^2 \right). \quad (22)$$

The common feature of these models is that the  $Q^2$  dependence disappears as  $x \rightarrow 1$ , even though the falloff in  $Q^2$  is very rapid for fixed  $x$ . For specific choices of  $f(x)$  one can thus accomplish  $F(Q^2) \sim 1/Q^2$  asymptotically — even if  $H(x, 0, Q^2)$ , for  $x$  fixed, falls off more rapidly with  $Q^2$ .

In order to illustrate possible consequences of this interplay between  $x$  and  $Q^2$  dependences we will discuss the specific class of exponential models. However, we emphasize that the conclusions are not tied to the specific functional dependence chosen, but are obtained for all models in which  $H(x, 0, Q^2)$  falls faster than  $1/Q^2$  at fixed  $x$ , with  $F(Q^2) \sim 1/Q^2$  from quarks with  $x \rightarrow 1$ .

Integrating  $H(x, 0, Q^2)$  of Eq. (22) over  $x$  yields a power law behavior for the form factor

$$F(Q^2) \sim \left( \frac{1}{Q^2} \right)^{\frac{m}{n}}. \quad (23)$$

However,  $m$  and  $n$  cannot be arbitrary. In order for the pion to have a finite size [i.e. integral in Eq. (9) converges], we must have

$$(m-2)/n > 0 \quad (24)$$

Consistency with the observed pion form factor ( $F \sim \frac{1}{Q^2}$ ) requires  $m = n$ . Therefore we consider

$$H(x, 0, Q^2) \xrightarrow{x \rightarrow 1} (1-x)^{n-1} \exp \left( -a(1-x)^n Q^2 \right), \quad (n > 2). \quad (25)$$

Straightforward application of Eq. (9) yields

$$b^2(Q^2)F(Q^2) \sim \int dx (1-x)^{2n-3} \exp \left( -a(1-x)^n Q^2 \right) \sim \left( \frac{1}{Q^2} \right)^{\frac{2n-2}{n}}. \quad (26)$$

Together with  $F(Q^2) \sim 1/Q^2$  we thus obtain

$$b^2(Q^2) \sim \left(\frac{1}{Q^2}\right)^{1-\frac{2}{n}}, \quad (27)$$

i.e. color transparency occurs only for  $n > 2$ . The case  $n < 2$  again corresponds to a situation where the effective size of the pion increases with  $Q^2$ . For  $n = 2$  the effective size of the pion approaches a finite limit for  $Q^2 \rightarrow \infty$ . In both cases ( $n < 2$  and  $n = 2$ ) there is no color transparency.

Quark counting rules predict  $q(x) \sim (1-x)^{n-1}$  with  $n = 2$  for the pion. This leaves several possibilities:

1. quark counting rules are correct but the piece in  $H(x, 0, Q^2)$  that describes the large  $Q^2$  behavior falls off faster at  $x \rightarrow 1$  than  $(1-x)$  [i.e.  $n > 2$ ]. In this case we should observe color transparency under suitable experimental conditions.
2. quark counting rules are correct and the same term in  $H(x, 0, Q^2)$  that describes the large  $Q^2$  behavior also describes the  $x \rightarrow 1$  behavior. In this case,  $n = 2$  in Eq. (27) and there would be no color transparency.
3. quark counting rules for the PDF are violated and the quark distribution function of the pion vanishes less rapidly than  $(1-x)$ , i.e.  $n < 2$  in Eq. (25). In this case there is no color transparency — in fact, in this bizarre case one would even observe anti-transparency (increasing cross section with  $Q^2$ ).

### 4.3 Models Based on Wave Functions

Factorized models of GPDs do not seem to arise naturally from simple dynamical assumptions[20, 21]. Here we assume a system made of two constituents of mass  $m$ . Then rotational invariance takes the form that the wave function is a function of  $\frac{k_\perp^2 + m^2}{x(1-x)}$ , where  $k_\perp$  is a relative momentum and the GPD is given by

$$H(x, 0, Q^2) = \frac{2m}{x(1-x)} \int d^2 k_\perp \psi^*(x, \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp) \psi(x, \mathbf{k}_\perp), \quad (28)$$

or

$$H(x, 0, \mathbf{B}) = \frac{2m}{x(1-x)^3} \psi^2(x, \frac{B^2}{(1-x)^2}). \quad (29)$$

The color transparency aspect of these kinds of models were discussed for the spin-less case in[7]. Here we present the case with spin included. We use the pionic model of Chung, Coester and Polyzou[22] as a starting point. In that model the form factor is given by

$$F_\pi(Q^2) = \frac{1}{4\pi} \int \frac{dx}{x(1-x)} \int d^2 k_\perp [k_\perp^2 + m^2 + (1-x)\mathbf{k}_\perp \cdot \mathbf{Q}] \phi(\mathbf{k}_\perp'^2) \phi(\mathbf{k}_\perp^2), \quad (30)$$

$$\phi(\mathbf{k}_\perp^2) = u(k^2)/[(\mathbf{k}_\perp^2 + m^2)/x(1-x)]^{1/4}, \quad k^2 = (\mathbf{k}_\perp^2 + m^2)/(4x(1-x)) - m^2, \quad (31)$$

$$\mathbf{k}' = \mathbf{k} + (1-x)\mathbf{Q}. \quad (32)$$

The function  $u$  is a solution to the wave equation:

$$(4(k^2 + m^2) + 4mV)u = M^2 u, \quad (33)$$

with  $M$  as the pion mass. Chung et al found that using a Gaussian form

$$u_G(k^2) = (4/\sqrt{\pi}b^3)^{1/2} \exp(-k^2/2b^2), \quad (34)$$



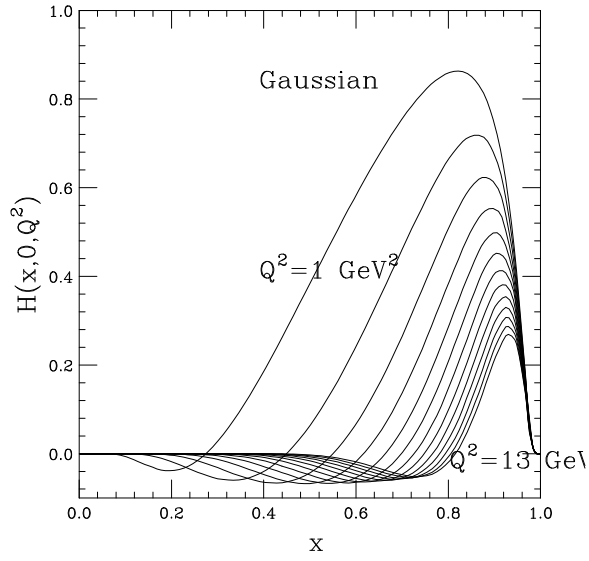


Figure 1: GPD for Gaussian model

with  $m = 0.21$  GeV and  $b = 0.35$  GeV gave a good description of the form factor. We shall also use a power law form[23]

$$u_P(k^2) = 1/(k^2 + b_p^2)^2 \sqrt{32b_p^5/\pi}, \quad (35)$$

with  $m = 0.21$  GeV and  $b_p = 0.4$  GeV, that gives a rather similar form factor.

In the present models the GPD is given by

$$H(x, 0, Q^2) = \frac{1}{4\pi x(1-x)} \int d^2 k_\perp [k_\perp^2 + m^2 + (1-x)\mathbf{k}_\perp \cdot \mathbf{Q}] \phi(\mathbf{k}_\perp'^2) \phi(\mathbf{k}_\perp^2) \quad (36)$$

The results are displayed in Figs. 1, 2. There is a clear tendency for the peak in  $x$  to move to increasing values as  $Q^2$  is increasing. This means the mechanism becomes more like that of Feynman. This trend is also seen in the computed values of  $b^2(Q^2)$  which increase with  $b^2$  as shown in Figs.3.



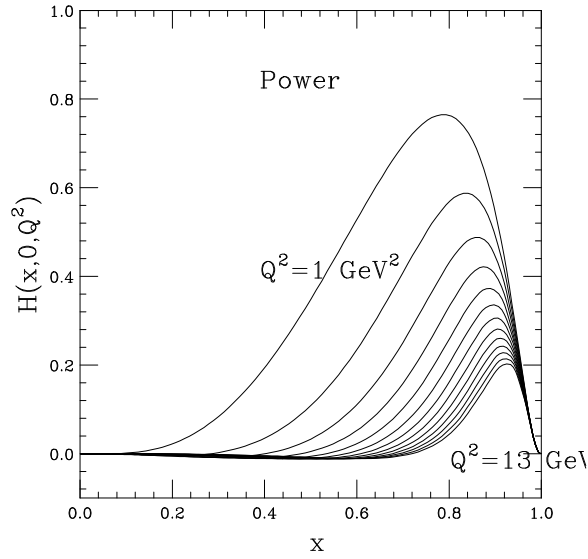


Figure 2: GPD for power law model

These results are obtained for simple models, and lead to conclusions in sharp contrast with experimental observations[12, 13]. However, they do provide a basis for obtaining a semi-analytic understanding of  $b^2(Q^2)$ . This can be seen by applying the mean value theorem to the integral over  $x$  of Eq.(8).

$$b^2(Q^2)F(Q^2) \approx -\frac{\partial}{\partial Q^2} \frac{1}{(1 - \bar{x}(Q^2))^2} F(Q^2) \quad (37)$$

$$b^2(Q^2) \approx \frac{2}{(1 - \bar{x}(Q^2))^3} \frac{d\bar{x}}{dQ^2} + \frac{1}{Q^2}. \quad (38)$$

If  $\bar{x}$  is large and increasing then the first term of Eq.(38) will be large and positive. Then  $b^2(Q^2)$  will be large and color transparency would be precluded. From Figs. 1, 2 it is clear with that such is the case for those models. With the discovery of color transparency for pions at our disposal, we can say that these models are ruled out even though they provide form factors in excellent agreement with experiment.

In general, precise statements can be made if one knows how  $\bar{x}(Q^2)$  approaches 1 for  $Q^2 \rightarrow \infty$ . The crucial question is whether

$$[1 - \bar{x}(Q^2)] Q \quad (39)$$

becomes infinite or not as  $Q^2 \rightarrow \infty$ . Let us first consider the case

$$\lim_{Q^2 \rightarrow \infty} Q [1 - \bar{x}] = 0. \quad (40)$$

This happens for example the case when the main  $Q^2$  dependence in  $H(x, 0, Q^2)$  near  $x = 1$  is through the combination  $(1-x)Q^2$ . An explicit example is provided by a dependence of the form  $H(x, 0, Q^2) \sim q(x) \exp[-a(1-x)Q^2]$  or  $H(x, 0, Q^2) \sim q(x) \exp[-a\frac{(1-x)}{x}Q^2]$ . In this example,  $[1 - \bar{x}(Q^2)] \sim \frac{1}{Q^2}$  and application of Eq. (38) to  $[1 - \bar{x}(Q^2)] \sim \frac{1}{Q^2}$  yields  $b^2(Q^2) \sim Q^2$  and therefore the effective size of the hadron grows with increasing  $Q^2$ .

The opposite happens when

$$\lim_{Q^2 \rightarrow \infty} Q [1 - \bar{x}] = \infty. \quad (41)$$

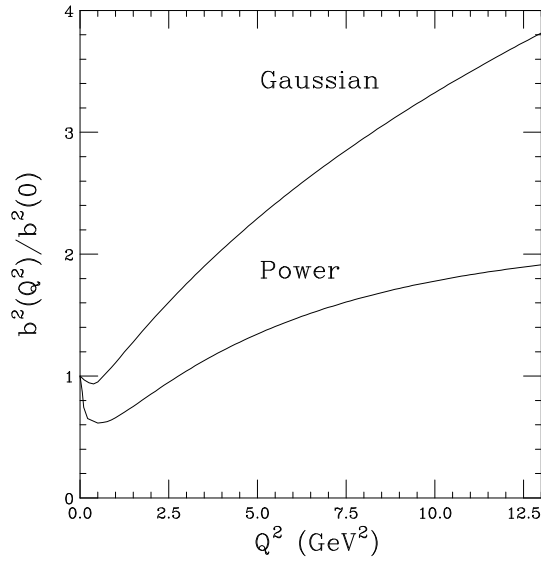


Figure 3:  $b^2(Q^2)$  for Gaussian and power models

An explicit example would be a dependence of the form  $H(x, 0, Q^2) \sim q(x) \exp[-a(1-x)^3 Q^2]$  near  $x \rightarrow 1$ , and thus  $[1 - \bar{x}(Q^2)] \sim \frac{1}{Q^{2/3}}$ . Inserting this result into Eq. (38) yields  $b^2(Q^2) \sim Q^{-2/3}$ , i.e. the effective size shrinks with increasing  $Q^2$ .

Finally, the marginal case corresponds to

$$\lim_{Q^2 \rightarrow \infty} Q [1 - \bar{x}] = \text{finite}. \quad (42)$$

An explicit example is given by  $H(x, 0, Q^2) \sim q(x) \exp[-a(1-x)^2 Q^2]$  near  $x \rightarrow 1$ , which yields  $[1 - \bar{x}(Q^2)] \sim \frac{1}{Q}$ . In this case

$$\lim_{Q^2 \rightarrow \infty} b^2(Q^2) = \text{const.} \quad (43)$$

## 5 Discussion

This paper is concerned with the relation between the effective size of a hadron  $b^2(Q^2)$  and generalized parton distributions,  $H(x, 0, Q^2)$ . A small effective size is a necessary condition for color transparency to occur. While a small value of  $b^2(Q^2)$  is a necessary consequence of perturbative QCD for large enough values of  $Q^2$ , the experimental limitation non-asymptotic values of  $Q^2$  causes studies of models of strongly interacting QCD to be relevant and interesting.

Our procedure is to examine a set of model pion wave functions, each giving rise to a form factor that falls as  $1/Q^2$  at large  $Q^2$ . This provides a set of examples to illustrate general principles that control the behavior of  $b^2(Q^2)$  at large  $Q^2$ . These can be summarized as follows

- If the GPD-representation for the form factor,  $H(x, 0, Q^2)$ , is dominated by the average value of  $x$  (see Eq. (38),  $\bar{x} \neq 1$  at large  $Q^2$  (e.g. PQCD mechanism) then the effective size  $b^2(Q^2)$  goes to zero at large  $Q^2$ . Then color transparency is expected to occur under suitable experimental conditions.
- On the other hand, if the GPD representation of the form factor is dominated by  $\bar{x} \rightarrow 1$  (e.g. Feynman mechanism) then the crucial question is how rapidly does  $\bar{x}$  approach 1 with increasing  $Q^2$ .
- The effective size of the hadron goes to zero at large  $Q^2$  (a necessary condition for color transparency) only if  $\lim_{Q^2 \rightarrow \infty} [1 - \bar{x}(Q^2)] Q = \infty$ .

The fact that color transparency has been observed [12, 13] indicates that the piece in  $H(x, 0, Q^2)$  that describes the large  $Q^2$  behavior does fall off faster at  $x \rightarrow 1$  than  $(1 - x)$ . In the notation of Eq. (25),  $n > 2$ . More generally, we can be confident that  $\lim_{Q^2 \rightarrow \infty} [1 - \bar{x}(Q^2)] Q = \infty$ . This means that either its GPD at fixed  $x$  falls off like the form factor, or the GPD which describes the leading behavior of the form factor vanishes near  $x \rightarrow 1$  faster than  $(1 - x)$ .

Our results provide a test for GPDs, and also a further test for light-cone wave functions of the pion. The simple pion wave function models used here are not consistent with the dependence on  $x$  required to obtain a small  $b^2(Q^2)$ , and we consider these to be ruled out. But there are a variety of models in the literature[24], and we suggest that  $b^2(Q^2)$  be evaluated to provide further tests.

Experimental studies of pionic color transparency are planned[25]. If color transparency is observed at momentum transfers accessible to Jefferson Laboratory, then our results provide predictions regarding the pionic GPD that could be tested by (difficult, but not impossible) experiments.

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